

Direct reconstruction of isosinglet amplitudes for nucleon-nucleon elastic scattering

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Abstract. The direct reconstruction of the isospin $I = 0$ amplitudes is discussed under the assumption that the $I = 1$ amplitudes are known and that a sufficient number of independent np observables have been measured at a centre-of-mass angle θ . We show that at least one observable measured at angle $\pi - \theta$ is necessary in order to determine the $I = 0$ amplitudes, including the phase relative to the $I = 1$ amplitudes. Special cases at $\theta = 0$, $\pi/2$ and π are also discussed.

1 Introduction

The present paper is devoted to the direct reconstruction of the elastic scattering amplitudes for the isospin $I = 0$ state using known pp and np amplitudes and observables. The direct reconstruction is a complementary method to the phase shift analysis (PSA). A PSA is able to reconstruct amplitudes from an incomplete set of data compensating the lack of observables by smooth angular dependences and model-dependent ingredients, e.g. one-pion exchange (OPE). The model-dependent part becomes more important with growing energy. A PSA provides the absolute phases of the amplitudes at all scattering angles θ , fixed by OPE contributions for long-range (peripheric) interactions. Therefore, the PSA procedure automatically determines the phase between the pp and np complex amplitudes. The PSA takes into account contributions of electromagnetic interactions and determines pure nuclear or total amplitudes. The isosinglet amplitudes can then be directly found.

This is not the case in the direct reconstruction of amplitudes. A direct amplitude reconstruction is carried out at a given angle and energy where it requires a complete set of data. The importance of the method is in fact that it is entirely model independent whatever the energy may be. It therefore provides an important check of the PSA. Via the reconstruction, measured observables for pp or np scattering provide the absolute values of amplitudes

and their relative phases for the respective systems. However, the $I = 0$ amplitudes are still not determined. The $I = 1$ amplitudes from pp scattering contain electromagnetic contributions which must be subtracted. In addition there is an undetermined relative phase between the pp and np amplitudes. To find this relative phase, which enables calculation of the isosinglet amplitudes, it is necessary to measure at least one np experiment at the angle $\pi - \theta$ as discussed in this paper.

A direct amplitude reconstruction is possible only if the pp and np data base is sufficient or ‘complete’, i.e. if at least 9 spin-dependent quantities and the differential cross section have been measured at one energy and angle. Up to now, all direct reconstructions of np scattering amplitudes at a centre-of-mass (c.m.) angle θ have been performed without consideration of the connections with observables at the conjugate angle $\pi - \theta$. We do not address the more general question of np amplitude reconstruction from two ‘incomplete’ sets of experiments, one at θ , the other at $\pi - \theta$. Interesting ideas on the subject may be found in [1].

Since 1990 abundant pp data from experiments performed at PSI, LAMPF and SATURNE II have been available. Direct $I = 1$ amplitude reconstructions have been performed [2–4] at several energies and over a large range of scattering angles.

The np situation has improved in recent years, in particular since the mid-1980’s when polarized neutron beams became available at a number of laboratories. This allowed measurements of spin-dependent observables in free elastic

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np scattering. The np differential cross sections are fairly well known below 0.8 GeV. Unfortunately at higher energies, measurements at c.m. angles between 30° and 120° are almost non-existent; accurate results exist only in the small angle and large angle regions. Therefore, even the $d\sigma/d\Omega$ data sets required for amplitude reconstructions are incomplete. Direct np amplitude reconstructions which are possible at various energies and angles or through proposed new measurements are summarized below:

- Analyses of SATURNE data at five energies from 0.84 to 1.1 GeV and at three or four c.m. angles less than 90° have been completed [5].
- Combined LAMPF and SATURNE II results form an overdetermined set of np observables at 0.8 GeV for two angles [5].
- New accurate PSI data in the energy region from 0.260 to 0.535 GeV have been measured in the angular range 60° – 160° [6]. These data will permit np amplitude analyses over most of that angular region.
- The first $\Delta\sigma_L(np)$ experiments above 1.1 GeV have been performed at the DUBNA synchrotron-nuclotron complex using a new polarized target [7]. Measurements of spin dependent np observables close to 180° up to 4 GeV are foreseen. These are needed for forward angle amplitude analysis.
- Spin-dependent total cross section data which permit a partial amplitude reconstruction are now available at 16.2 MeV [8].

In this paper we discuss in detail the requirements for an unambiguous determination of $I = 0$ amplitudes by a direct amplitude reconstruction. The NN amplitude formalism is reviewed in Sect. 2, Sect. 3 discusses amplitude reconstruction in the forward direction, and Sect. 4 handles the reconstruction for the general case.

2 The NN scattering matrix and amplitude symmetries

The nucleon-nucleon elastic scattering formalism, amplitude representation and four-spin-index notation from [9] are used throughout this paper. Assuming parity conservation, time reversal invariance and isospin invariance, we write the scattering matrix in the form

$$M(\vec{k}_f, \vec{k}_i) = \frac{1}{2}[(a+b) + (a-b)(\vec{\sigma}_1 \cdot \vec{n})(\vec{\sigma}_2 \cdot \vec{n}) + (c+d)(\vec{\sigma}_1 \cdot \vec{m})(\vec{\sigma}_2 \cdot \vec{m}) + (c-d)(\vec{\sigma}_1 \cdot \vec{l})(\vec{\sigma}_2 \cdot \vec{l}) + e(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{n}], \quad (2.1)$$

where a, b, c, d and e are five complex scattering amplitudes which are functions of energy and c.m. scattering angle θ . As the energy is constant throughout the present article, it is omitted. $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli 2×2 matrices, \vec{k}_i and \vec{k}_f are unit vectors in the direction of the incident and scattered particles, respectively, and

$$\vec{n} = \frac{(\vec{k}_i \times \vec{k}_f)}{|\vec{k}_i \times \vec{k}_f|}, \quad \vec{l} = \frac{(\vec{k}_f + \vec{k}_i)}{|\vec{k}_f + \vec{k}_i|}, \quad \vec{m} = \frac{(\vec{k}_f - \vec{k}_i)}{|\vec{k}_f - \vec{k}_i|}. \quad (2.2)$$

Table 1. Symmetry properties of the NN scattering amplitudes

$I = 0$ amplitudes	$I = 1$ amplitudes
$a_0(\theta) = a_0(\pi - \theta)$	$a_1(\theta) = -a_1(\pi - \theta)$
$b_0(\theta) = c_0(\pi - \theta)$	$b_1(\theta) = -c_1(\pi - \theta)$
$c_0(\theta) = b_0(\pi - \theta)$	$c_1(\theta) = -b_1(\pi - \theta)$
$d_0(\theta) = -d_0(\pi - \theta)$	$d_1(\theta) = d_1(\pi - \theta)$
$e_0(\theta) = -e_0(\pi - \theta)$	$e_1(\theta) = e_1(\pi - \theta)$

We can write the scattering matrices for pp , np and nn as

$$M(\vec{k}_f, \vec{k}_i) = \frac{M_0}{4}[1 - (\vec{\tau}_1 \cdot \vec{\tau}_2)] + \frac{M_1}{4}[3 + (\vec{\tau}_1 \cdot \vec{\tau}_2)] \quad (2.3)$$

where the isosinglet and isotriplet scattering matrices, M_0 and M_1 , respectively, each have the form given in (2.1). $\vec{\tau}_1$ and $\vec{\tau}_2$ are the nucleon isospin matrices. The corresponding amplitudes a to e acquire an isospin index 0 or 1. More specifically we have

$$M(pp \rightarrow pp) = M(nn \rightarrow nn) = M_1 \quad (2.4a)$$

$$M(np \rightarrow np) = M(pn \rightarrow pn) = (M_1 + M_0)/2 \quad (2.4b)$$

$$M(np \rightarrow pn) = M(pn \rightarrow np) = (M_1 - M_0)/2 \quad (2.4c)$$

As well as giving (2.4c), the generalized Pauli principle for nucleons also yields other symmetry conditions for the amplitudes a to e which connect amplitudes for a given isospin state at angles θ and $\pi - \theta$. These are summarized in Table 1. Obtaining the forward-backward relation between the amplitudes b and c is not immediately obvious. [9] gives the relations between the various amplitudes used in many different amplitude representations. The relations between amplitudes b and c are most easily seen when one takes the spin singlet-triplet representation of the scattering matrix, rewrites those amplitudes using the $\{a, b, c, d, e\}$ representation given in (2.1) and applies the results expressed in (2.4b) and (2.4c).

3 Isosinglet amplitude reconstruction: forward angles

The scattering matrix simplifies at forward ($\theta = 0$) and backward ($\theta = \pi$) angles. The amplitudes in (2.1) then satisfy

$$e(0) = 0, \quad a(0) - b(0) = c(0) + d(0) \quad (3.1a)$$

$$e(\pi) = 0, \quad a(\pi) - b(\pi) = c(\pi) - d(\pi) \quad (3.1b)$$

The subscripts for the amplitudes are omitted as the equations apply for the general NN case. Three complex amplitudes remain for each of pp and np scattering. Three independent total cross sections may be measured in the forward direction by pp and np transmission experiments

$$\sigma_{\text{tot}} = \sigma_{0\text{tot}} + \sigma_{1\text{tot}}(\vec{P}_B \cdot \vec{P}_T) + \sigma_{2\text{tot}}(\vec{P}_B \cdot \vec{k}_i)(\vec{P}_T \cdot \vec{k}_i) \quad (3.2)$$

where \vec{P}_B and \vec{P}_T are the beam and target polarizations and $\sigma_{0\text{tot}}$ is the spin-independent total cross section. The terms $\sigma_{1\text{tot}}$ and $\sigma_{2\text{tot}}$ are related to the total cross section differences $\Delta\sigma_T$ and $\Delta\sigma_L$, measurable with appropriately polarized initial nucleons. They are linear functions of three independent non-vanishing amplitude combinations via the optical theorems:

$$\sigma_{\text{tot}} = (2\pi/k)\Im m[a(0) + b(0)] \quad (3.3a)$$

$$-\Delta\sigma_T = 2\sigma_{1\text{tot}} = (4\pi/k)\Im m[c(0) + d(0)] \quad (3.3b)$$

$$\begin{aligned} -\Delta\sigma_L &= 2(\sigma_{1\text{tot}} + \sigma_{2\text{tot}}) \\ &= (4\pi/k)\Im m[c(0) - d(0)]. \end{aligned} \quad (3.3c)$$

Values of $\Im m[a(0) + b(0)]$ for $I = 0$ can be deduced from the known spin-independent pp and np total cross sections. The imaginary parts of the spin-dependent amplitudes $\Im mc(0)$ and $\Im md(0)$ for $I = 0$ were determined [10] at PSI (8 energies) and at SATURNE II (10 energies). In this case, $\Delta\sigma_T$ and $\Delta\sigma_L$ for pp data were fitted by PSA and the corresponding measured np data were used.

To reconstruct the real parts of the forward scattering amplitudes one needs to determine at least three other observables for pp as well as for np scattering. No *scattering* observable can actually be measured at exactly 0° , in fact no np observables have been measured at angles smaller than 10° [10].

For the pp system data are available as low as 4° . Therefore the nuclear real parts of the pp forward amplitudes can be reliably extrapolated to $\theta = 0^\circ$ using a PSA procedure, which represents the best fit to the angular dependence of any measured observable.

For the np system the use of forward observables may be replaced by measurements of the differential cross section and two non-vanishing two-spin parameters at $\theta = \pi$. In the intermediate energy domain the best choice of observables includes spin correlation parameters since they are large in the backward angle region.

As an example, consider the observables $A_{oonn}(\pi)$ and $A_{ookk}(\pi)$ provided by PSI and SATURNE II measurements [11]. Since $e(0) = e(\pi) = 0$, we have

$$\frac{d\sigma}{d\Omega}(\pi) = \frac{1}{2}(|a|^2 + |b|^2 + |c|^2 + |d|^2) \quad (3.4)$$

$$\frac{d\sigma}{d\Omega}A_{oonn}(\pi) = \frac{1}{2}(|a|^2 - |b|^2 - |c|^2 + |d|^2) \quad (3.5)$$

$$\frac{d\sigma}{d\Omega}A_{ookk}(\pi) = \Re e a^*d + \Re e b^*c \quad (3.6)$$

Using simple relations of the type

$$|a + d|^2 = |a|^2 + |d|^2 + 2\Re e a^*d \quad (3.7)$$

together with (3.1b) we find

$$\frac{d\sigma}{d\Omega}(1 + A_{ookk}) = |b + c|^2 \quad (3.8)$$

$$\frac{d\sigma}{d\Omega}(1 - A_{ookk} - 2A_{oonn}) = |b - c|^2 \quad (3.9)$$

$$\frac{d\sigma}{d\Omega}(1 - A_{ookk} + 2A_{oonn}) = |2d - b - c|^2 \quad (3.10)$$

where all quantities are calculated at $\theta = \pi$. Using the specified experiments the three remaining real parts of the np amplitudes are obtained, but with independent sign ambiguities for each amplitude combination and, therefore, we get eight possible solutions.

The ambiguities may be removed in several different manners. First, the sign of the ratio

$$\rho = \Re e(a(0) + b(0))/\Im m(a(0) + b(0)) \quad (3.11)$$

of pp and np forward scattering amplitudes known from measurements at small angles may sometimes be used. Any independent experiment measured at $\theta = \pi$ decreases the number of ambiguities. Supplementary observables were used in this manner in [11] where the np observables K_{onno} $K_{ok''ko}$ could be fairly well estimated at $\theta = \pi$ below 0.6 GeV. At higher energies other observables may be more easily accessible.

4 Isosinglet amplitude reconstruction: general case

Let us express the np scattering amplitudes at an angle θ in terms of the isospin 0 and 1 amplitudes (see (2.3)):

$$\begin{aligned} a_{np} &\equiv \frac{1}{2}(a_0 + a_1) \\ b_{np} &\equiv \frac{1}{2}(b_0 + b_1) \\ c_{np} &\equiv \frac{1}{2}(c_0 + c_1) \\ d_{np} &\equiv \frac{1}{2}(d_0 + d_1) \\ e_{np} &\equiv \frac{1}{2}(e_0 + e_1). \end{aligned} \quad (4.1)$$

Using Table 1 we find the following relations between the np amplitudes at angle $\pi - \theta$ and the np and pp amplitudes at angle θ :

$$\begin{aligned} a_{np}(\pi - \theta) &= a_{np} - a_1 \\ b_{np}(\pi - \theta) &= c_{np} - c_1 \\ c_{np}(\pi - \theta) &= b_{np} - b_1 \\ d_{np}(\pi - \theta) &= -d_{np} + d_1 \\ e_{np}(\pi - \theta) &= -e_{np} + e_1. \end{aligned} \quad (4.2)$$

Suppose complete sets of pp and np elastic scattering observables have been measured at the angle θ and the scattering amplitudes for both systems have been directly reconstructed, each up to an overall phase. Assume that in both cases the overall phase was the phase of the corresponding amplitude e , denoted by ε_1 for pp and ε_{np} for np scattering. This means that for each amplitude we know its absolute value and its phase relative to e_1 or e_{np} , e.g. for the amplitude a_1 we have determined $|a_1|$ and $\alpha_1 - \varepsilon_1$. To reconstruct the isosinglet amplitudes we have to find

the relative phase $\varepsilon_{np} - \varepsilon_1$ between the pp and the np amplitudes.

We will show that to find $\varepsilon_{np} - \varepsilon_1$ with one discrete ambiguity we need to measure one np experiment at the angle $\pi - \theta$. Two such experiments will provide $\varepsilon_{np} - \varepsilon_1$ unambiguously.

Denote

$$x \equiv \tan \frac{\varepsilon_{np} - \varepsilon_1}{2} \quad (4.3)$$

Then

$$\sin(\varepsilon_{np} - \varepsilon_1) = \frac{2x}{1+x^2} \quad (4.4)$$

and

$$\cos(\varepsilon_{np} - \varepsilon_1) = \frac{1-x^2}{1+x^2}. \quad (4.5)$$

As a first example for finding $\varepsilon_{np} - \varepsilon_1$ let us consider the np differential cross-section at the angle $\pi - \theta$. Using (4.2) this may be written as

$$\begin{aligned} \frac{d\sigma_{np}}{d\Omega}(\pi - \theta) &= \frac{1}{2}(|a_{np} - a_1|^2 + |b_{np} - b_1|^2 \\ &\quad + |c_{np} - c_1|^2 + |d_{np} - d_1|^2 + |e_{np} - e_1|^2) \\ &= \frac{d\sigma_{np}}{d\Omega}(\theta) + \frac{d\sigma_{pp}}{d\Omega}(\theta) - \Re(a_{np}a_1^* + b_{np}b_1^* \\ &\quad + c_{np}c_1^* + d_{np}d_1^* + e_{np}e_1^*). \end{aligned} \quad (4.6)$$

To isolate the isosinglet-isotriplet phase difference and express quantities in terms of the individual reconstructed pp and np amplitudes and phases it is convenient to write

$$\Re a_{np}a_1^* = |a_{np}| |a_1| \cos(\alpha_{np} - \alpha_1). \quad (4.7)$$

We can then expand

$$\begin{aligned} \cos(\alpha_{np} - \alpha_1) &= \cos(\alpha_{np} - \varepsilon_{np} - (\alpha_1 - \varepsilon_1) + \varepsilon_{np} - \varepsilon_1) \\ &= \cos(\alpha_{np} - \varepsilon_{np} - (\alpha_1 - \varepsilon_1)) \cos(\varepsilon_{np} - \varepsilon_1) \\ &\quad - \sin(\alpha_{np} - \varepsilon_{np} - (\alpha_1 - \varepsilon_1)) \sin(\varepsilon_{np} - \varepsilon_1) \\ &= \cos(\alpha_{np} - \varepsilon_{np} - (\alpha_1 - \varepsilon_1)) \frac{1-x^2}{1+x^2} \\ &\quad - \sin(\alpha_{np} - \varepsilon_{np} - (\alpha_1 - \varepsilon_1)) \frac{2x}{1+x^2} \end{aligned} \quad (4.8)$$

and similar expressions for other real parts in (4.6). This will transform (4.6) to a quadratic equation in x from which one obtains $\varepsilon_{np} - \varepsilon_1$ with one discrete ambiguity.

Instead of the np differential cross section one may measure any other observable O_{np} at $\pi - \theta$. Indeed, using (4.6–4.8) we may write

$$\begin{aligned} \frac{d\sigma_{np}(\pi - \theta)}{d\Omega} O_{np}(\pi - \theta) &= \left[\frac{d\sigma_{np}}{d\Omega}(\theta) + \frac{d\sigma_{pp}}{d\Omega}(\theta) + C_1 \frac{1-x^2}{1+x^2} + C_2 \frac{2x}{1+x^2} \right] \\ &\quad \times O_{np}(\pi - \theta). \end{aligned} \quad (4.9)$$

A second expression for $\frac{d\sigma}{d\Omega} O_{np}$ is obtained by writing it directly as a quadratic form of the scattering amplitudes

$$\begin{aligned} \frac{d\sigma_{np}(\pi - \theta)}{d\Omega} O_{np}(\pi - \theta) &= C_3 + C_4 \frac{1-x^2}{1+x^2} \\ &\quad + C_5 \frac{2x}{1+x^2}. \end{aligned} \quad (4.10)$$

In these last two equations C_1, C_2, C_3, C_4 and C_5 are appropriate polynomials of the pp and np amplitudes known from the direct reconstruction at angle θ . Equating the right-hand sides of (4.9) and (4.10) we again get a quadratic equation for x from which it is easy to determine $\varepsilon_{np} - \varepsilon_1$ with at most one discrete ambiguity. Information from a second np experiment at $\pi - \theta$ will remove the ambiguity.

Note that the knowledge from any two different experiments measured at $\pi - \theta$ can be expressed as a system of two linear equations for $\cos(\varepsilon_{np} - \varepsilon_1)$ and $\sin(\varepsilon_{np} - \varepsilon_1)$ which may be solved instead of the two quadratic equations for x . In practice, however, the experimental values are subject to statistical errors and the method of least squares should be used to insure compatibility between the sin and cos values.

In the special case of $\theta = \pi/2$ we have $e_0(\pi/2) = 0$ and $2e_{np}(\pi/2) = e_1(\pi/2)$ from Table 1. In this case $\varepsilon_{np} - \varepsilon_1 = 0$ and a direct reconstruction of the pp and np amplitudes also provides the isospin $I = 0$ amplitudes without any additional measurement.

5 Conclusions

This paper gives for the first time a comprehensive prescription how to determine the relative phase of the $I = 0$ and $I = 1$ components of the NN scattering matrix at any angle in a direct amplitude reconstruction. So far, this subject has been treated incompletely because of the lack of adequate np data. In the past few years, extensive np scattering data have become available, renewing interest in the problem. The procedure requires that a complete set of observables be available for both the pp and np systems. The complete sets of experiments at an angle θ allow direct reconstruction of the pp and np scattering amplitudes independently, but are not sufficient to determine the pure isosinglet amplitudes. One extra np experiment at the angle $\pi - \theta$ is necessary to calculate the isosinglet amplitudes with one discrete ambiguity; two experiments at $\pi - \theta$ are sufficient to get an unambiguous solution. The knowledge of the isosinglet amplitudes then allows the np amplitudes at the angle $\pi - \theta$ to be determined.

Complete sets of observables which are appropriate for use in such a direct amplitude reconstruction are now available below 2.7 GeV pp kinetic energy and below 1.1 GeV for np scattering. Experiments are proposed at JINR-Dubna which may extend the energy ranges up to 4 GeV. Measurements in the forward direction check the validity of dispersion relations and at high energies the behaviour of amplitudes, as predicted by QCD, may be tested.

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References

1. H. Spinka, Phys. Rev. D **30** (1984) 1461
2. E. Aprile et al., Phys. Rev. Lett. **46** (1981) 1047; R. Hausammann et al., Phys. Rev. D **40** (1989) 22
3. M.W. McNaughton et al., Phys. Rev. C **41** (1990) 2809
4. C.D. Lac et al., J. Phys. (Paris) **51** (1990) 2689
5. J. Ball et al., Nuovo Cimento A **111** (1998) 13
6. N. Naef, Ph.D. Thesis #2832, University of Geneva (1996); A. Teglia, Ph.D. Thesis #2948, University of Geneva (1997); SPIN96, 12th International Symposium on High-Energy Spin Physics, edited by C.W. de Jager, T.J. Ketel, P.J. Muldres, J.E.J. Oberski and M. Oskam-Tamboezer (World Scientific, 1997) pp 588–590; PANIC96, 14th International Conference on Particles and Nuclei, edited by C.E. Carlson and J.J. Domingo (World Scientific, 1997) pp 343–344
7. B.P. Adiasovich et al., Zeitschrift für Physik **C71** (1996) 65
8. J. Brož et al., Zeitschrift für Physik **A354** (1996) 401; J. Brož et al., Zeitschrift für Physik **A359** (1997) 23
9. J. Bystrický et al., J. Phys. (Paris) **39** (1978) 1
10. C. Lechanoine-Leluc, F. Lehar, Rev. Mod. Phys. **65** (1993) 47
11. R. Binz, Ph.D. thesis, Freiburg University, Germany (1991)